Supersonic Boundary-Layer Transition: Effects of Roughness and Freestream Disturbances

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Boundary-layer transition experiments were conducted in the AEDC-VKF, 12- and 40-in. supersonic wind tunnels at Mach numbers 3 and 4 on an adiabatic wall, 10° total-angle sharp cone. The effects of spherical roughness elements and freestream disturbances (radiated aerodynamic noise) on transition were investigated. The large variation in smooth wall transition locations which exists on models in supersonic wind tunnels of different sizes is shown to influence trip performance. These studies have also indicated that the "effective point" location proposed by van Driest et al., is relatively independent of the smooth wall transition location (or freestream disturbance and tunnel size) at supersonic speeds. The correlation parameters developed by van Driest-Blumer and Potter-Whitfield are examined, and their ability to predict the tripped transition location is discussed.

Nomenclature

b = model nose bluntness, in.
C_F = mean turbulent skin-friction coefficient (tunnel wall)
c = tunnel test section circumference, in.
c_1 = tunnel test section circumference of 12- × 12-in. tunnel (c_1 = 48 in.)
k = diameter of roughness element (sphere diameter), in.
$\bar{k}$ = total height of roughness element above model surface (sphere diameter plus band thickness), in.
M = Mach number
Re_k = trip Reynolds number ($\rho_\delta U_\delta k/\mu_\delta$)
$Re_{\bar{k}}$ = Potter-Whitfield trip correlation Reynolds number

$$Re_{\bar{k}}' = [(u_{\bar{k}}\bar{k})/\nu_{\bar{k}}](T_{\bar{k}}/T_w)^{+0.5+\omega}$$

for adiabatic wall

$$Re_{\bar{k}}' = \bar{k}(U_\delta/\nu_\delta)(u_{\bar{k}}/U_\delta)(T_\delta/T_{\bar{k}})^{0.5}/1 + \tfrac{1}{2}(\gamma - 1)\eta_r M_\delta{}^2]^{0.5+\omega}$$
$$= \bar{k}(U_\delta/\nu_\delta)(M_{\bar{k}}/M_\delta)[1 + \tfrac{1}{2}(\gamma - 1) \times \eta_r M_\delta{}^2]^{-1.26}$$

$(Re_t)_\delta$ = transition Reynolds number (based on local conditions), $(Re_t)_\delta = (Re/\text{in.})_\delta(x_t) = U_\delta x_t/\nu_\delta$
Re_{xk} = trip position Reynolds number ($\rho_\delta U_\delta x_k/\mu_\delta$)
Re_δ, $(Re/\text{in.})_\delta$ = inviscid flow local surface unit Reynolds number per inch, U_δ/ν_δ
Re_∞, $(Re/\text{in.})_\infty$ = freestream Reynolds number per in., U_∞/ν_∞
$Re_{t_{\text{eff}}}$ = effective point transition Reynolds number ($\rho_\delta U_\delta x_{t_{\text{eff}}}/\mu_\delta$)
T = temperature, °R and/or °F
T_{DP} = dew point temperature at atmospheric pressure, °F
$(T_e/T_\delta)_{\text{corr}}$ = corrected effective temperature ratio as defined in Ref. 4
T_0 = tunnel stilling chamber total temperature, °R and/or °F
U = velocity outside the boundary layer, fps
u = velocity in the boundary layer, fps
x = surface distance measured from cone apex, in.
x_k = distance from cone tip to center of roughness sphere, in.
x_t = surface distance location of boundary-layer transition, in.
$x_{t_{\text{eff}}}$ = effective point transition location, in.
x_{t_o} = transition location on smooth body, in.
y = distance normal to surface, in.
γ = ratio of specific heats
δ^* = boundary-layer displacement thickness (tunnel wall), in.
ϵ = value of $Re_{\bar{k}}'$ where $x_t = x_k$; $\epsilon = f(M_{\bar{k}})$ as shown in Refs. 6–8
η_r = temperature recovery factor, $(T_{AW} - T_\delta)/T_o - T_\delta)$
θ_c = cone half angle, deg
μ = absolute viscosity, lb-sec/ft²
ν = kinematic viscosity, in-fps
ρ = density, lb-sec²/in.-ft³
ω = exponent in viscosity-temperature relation (ω = 0.76)

Subscripts

AW = adiabatic wall
c = cone configuration
k = at height k in undisturbed laminar boundary layer
$\bar{k}$ = at total height $\bar{k}$ in undisturbed laminar boundary layer
planar = two-dimensional configuration, either hollow cylinder or flat plate
W = wall
δ = edge of boundary layer
∞ = freestream

I. Introduction

THE literature provides abundant documentation of the many aerodynamic reasons for studying the boundary-layer transition process and the necessity of providing effective boundary-layer tripping mechanisms for application in wind tunnel experiments.[1–12] The effectiveness of surface roughness for inducing laminar boundary-layer transition on planar and conical bodies at supersonic and hypersonic speeds has been extensively reported in Refs. 1–10. Although there may remain considerable doubt about the effectiveness of surface roughness in promoting "early" transition at hypersonic speeds,[5–10] the evidence to date[1–6] would indicate that some confidence should be expected in estimating and predicting trip performance in the supersonic speed range ($M_\delta \lesssim 5$).

However, there may be some cause for concern when consideration is given to the absence of experimental data on the potential coupling between freestream disturbances at supersonic speeds ($M_\infty \gtrsim 3$) and the disturbance generated by the controlled surface roughness. This question becomes more pertinent in light of the recent results of Pate and Schueler[11]

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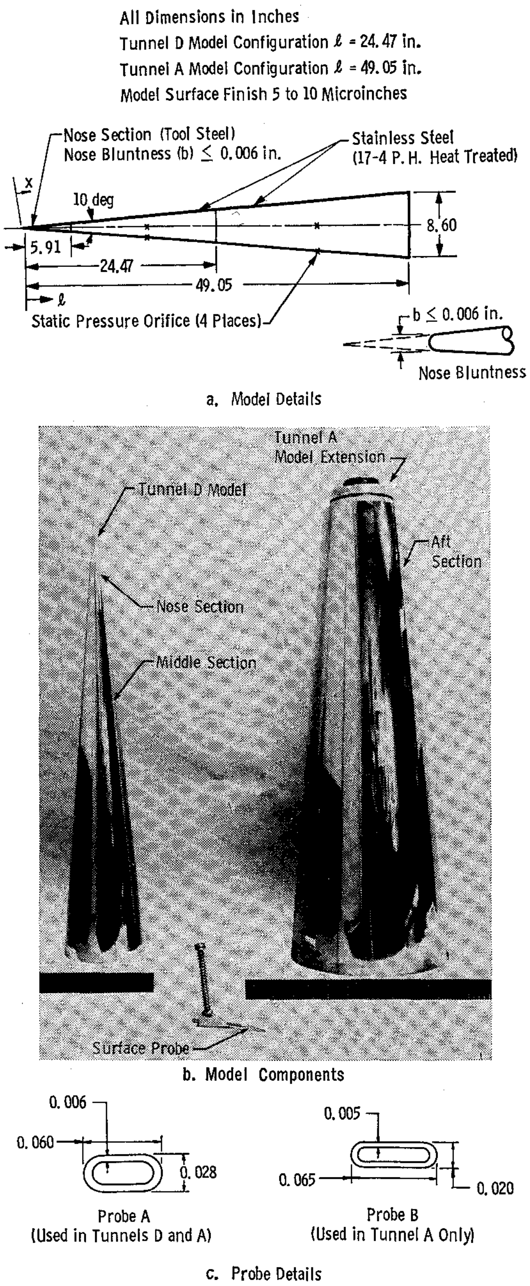

Fig. 1 Model geometry.

and Pate.[12] These results have shown the significant variations in smooth wall transition Reynolds numbers which exist between various size test facilities ($3 \lesssim M_\infty \lesssim 8$) as a result of the freestream disturbances present in the form of radiated aerodynamic noise.

This paper presents the experimental results of an investigation undertaken to determine if boundary-layer trip effectiveness is related to the tunnel size and the tunnel disturbance levels, per se. The studies were conducted in the VKF 12- and 40-in. supersonic wind tunnels at $M_\infty = 3$ and 4 on an adiabatic wall 10° total-angle sharp cone. Basic results are presented, analyzed, and compared with the two well-known and widely used trip correlations developed by van Driest-Blumer and Potter-Whitfield.

The methods of van Driest-Blumer and Potter-Whitfield were selected for comparison and analysis not only because of their presumed general usage, but also because of their basic differences. The method of van Driest-Blumer uses the more classical approach[1–5] of correlating trip height Reynolds numbers with a specific tripped transition Reynolds number. The Potter-Whitfield correlation provides values of the trip size required to locate transition anywhere between the smooth surface (no trip) transition location and the trip position.

II. Experimental Conditions

New experimental data included in this paper were obtained at the Arnold Engineering Development Center (AEDC) in the von Kármán Gas Dynamics Facility (VKF), Supersonic Tunnels A and D.

Tunnel D is an intermittent, variable density wind tunnel with a manually adjusted, flexible-plate-type nozzle and a 12- by 12-in. test section. The tunnel operates at Mach numbers from 1.5 to 5 at stagnation pressures from about 5 to 60 psia and average stagnation temperatures of about 70°F.

Tunnel A is a continuous, closed-circuit, variable-density wind tunnel with an automatically driven, flexible-plate-type nozzle and a 40- by 40-in. test section. The tunnel operates at Mach numbers from 1.5 to 6 at maximum stagnation pressures from 29 to 200 psia, respectively, and stagnation temperatures up to 290°F ($M_\infty = 6$). Minimum operating pressures range from about one-tenth to one-twentieth of the maximum pressures.

The transitional model, Figs. 1 and 2, was a 10° total-angle, stainless-steel cone equipped with a tool steel nose section. The model had a surface finish of 10 μin. (indicated by a surface analyzer) and a tip total bluntness (b) between 0.005 and 0.006 in. At the test conditions investigated the nose bluntness was sufficiently small to have negligible effects on the location of transition. The Tunnel D model consisted of the nose and middle section, as shown in Fig. 1. The Tunnel A model was obtained by adding an aft section as shown in Figs. 1a and 1b. To maintain a near perfect joint between the sections, it was necessary to refinish the model surface after attaching each model section.

The boundary-layer trip mechanism consisted of a single row of precision steel balls having diameters (k) of 0.010 and 0.015 in. and attached with epoxy resins to a $\frac{1}{4}$-in.-wide steel band having a thickness less than 0.002 in. as illustrated in Fig. 2. Ball spacing on the band was maintained constant at $\frac{1}{16}$ in. ($\approx 4k$) between centers, and the band centerline was positioned approximately 4.9 in. from the cone apex. The trip bands were machined to match the cone surface angle and were attached to the surface using very small amounts of glue. Variations in the two sets of ball heights above the band were approximately ±2 to 3%, and the average heights ($\bar{k}$) from the cone surface to the top of the 0.010- and 0.015-in.-diam spheres were 0.0117 and 0.0172 in., respectively.

It was stated in Ref. 3 that for test conditions at $M_\delta = 2.71$ similar to those of the present experiments a band height less than 0.002 in. had a negligible effect on the smooth wall transition data, and consequently the trip performance, providing the band was located 5 in. downstream of the cone tip.

A remotely controlled, electrically driven, surface pitot probe as shown in Figs. 1b and 3 provided a continuous trace of the probe pressure on an X-Y plotter from which the location of transition was determined. For this investigation the peak in the surface probe pressure trace (see Figs. 5 and 6) was selected to represent the location of transition. This method of detection is generally accepted as occurring near the end of the transition process (or zone). Tests in Tunnel A using different size probes established that probe size effects on the location of transition (x_{t_0}) were negligible. Schlieren photography was also utilized as a secondary method for detecting the location of transition in Tunnel D.

III. Results and Discussion

As mentioned in Sec. I, the purpose of this research was to determine if spherical roughness trips were equally effective in different size supersonic ($M_\infty = 3$ and 4) wind tunnels where the differences in freestream disturbances were suffi-

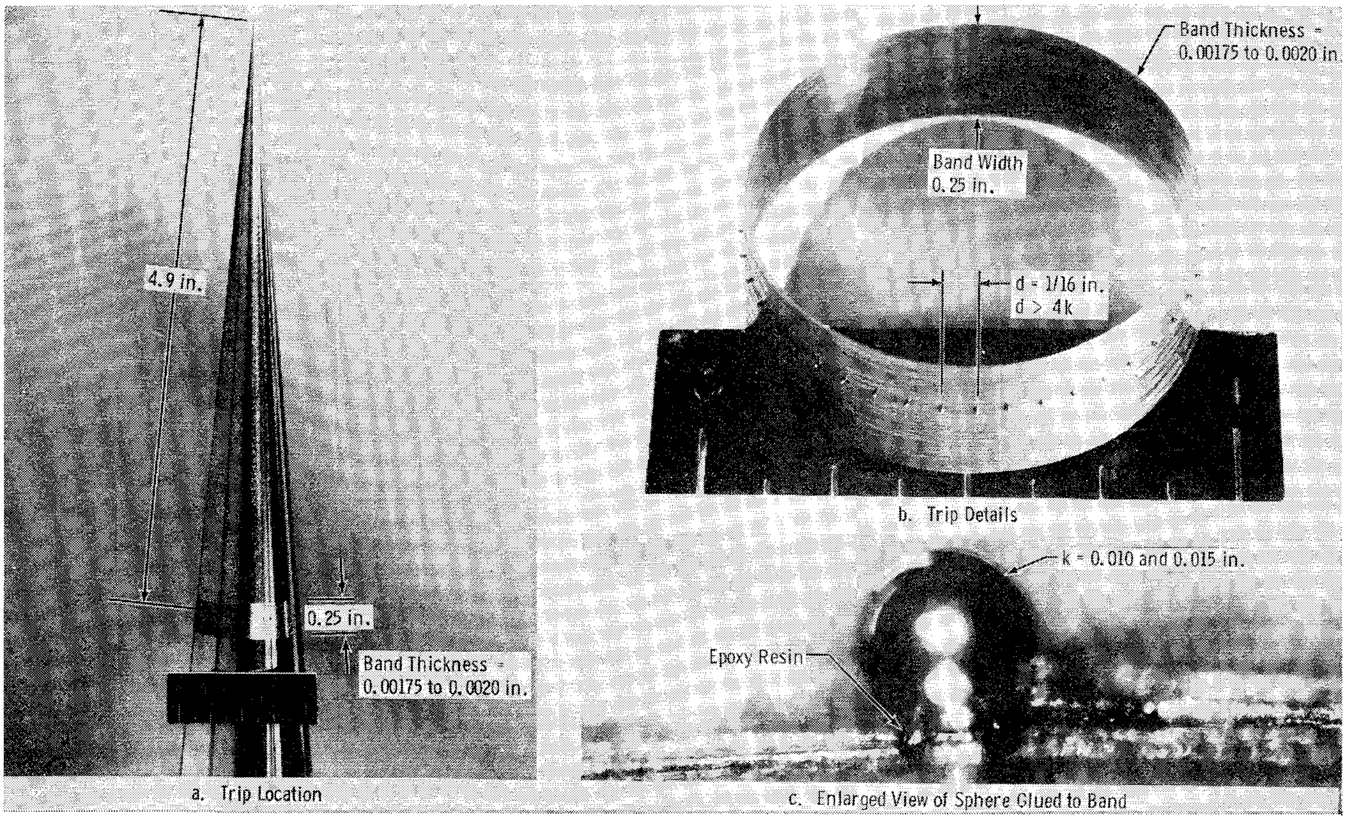

Fig. 2 Trip geometry.

cient to alter the smooth surface $(Re_t)_\delta$ values. Pate and Schueler[11] and Pate[12] have investigated the relationship between freestream disturbances which radiate from the tunnel wall turbulent boundary layer and their effect on the location of transition. Variations in $(Re_t)_\delta$ with tunnel size as a result of these radiated noise effects are very significant, as exemplified by the data presented in Fig. 4 for both planar and sharp cones. The cone data presented in Fig. 4b were obtained on the test model used for the current trip experiments, and these basic data will appear in different forms in later sections of this paper. Based on the aerodynamic noise dominance philosophy, extensive correlations of planar and sharp cone $(Re_t)_\delta$ data were developed in Refs. 11 and 12.

A series of basic transition profiles obtained with the traversing surface probe is presented in Figs. 5 and 6. These data show that the surface probe was able to detect and provide a distinct transition profile very near the trip position. The quality of the probe data taken at $M_\infty = 4$ in Tunnel D was comparable to the $M_\infty = 3$ traces shown in Fig. 5. Transition locations determined using the surface probe (with the x_t location selected at the pressure peak as shown in Figs. 5 and 6) were also consistent with the location of transition determined from Schlieren photographs. Additional comparison between probe and Schlieren results will be presented in the following section.

The various regions of interest which exist in a tripped transition-location profile are presented in Figs. 7a and 8. Region I illustrates the variations in the smooth surface transition location as a function of the tunnel unit Reynolds number. A trend of this form is quite common,[3] and the level (x_{to}) has recently been shown to increase significantly with increasing tunnel size, as discussed in the preceding section.

Region IA is dominated by the freestream disturbance levels, although trip disturbances are also being introduced into the transition process. Region II is a region of multiple dominance, and Region III (which represents the region between the "effective point" or "knee" and the trip) is dominated by the trip. The classification of the various parts of the tripped profile is taken from the definitions proposed by van Driest and Blumer.[3] The "effective point" is by definition[3] the point where the transition Reynolds number is a minimum as illustrated in Fig. 10. The author agrees with this interpretation of the phenomenon and chose to apply these criteria to the present study.

It is of interest to note that the Schlieren transition location results presented in Figs. 7 and 9 are about 10% to 20% lower than the probe locations which are near the end of the transition zone in Region I, but as the tripped values of x_t approach x_k, then the transition locations are about equal. One conclusion to be deduced from this observation is that the "effective point" locations when determined from conventional Schlieren photographs are not as distinguishable as the probe x_t locations.

Composite plots of the tripped data from Tunnels A and D from $M_\infty = 3$ and 4 are shown in Fig. 10 as a function of $(Re_t)_\delta$ and the trip Reynolds number (Re_k). Data presented in this form allow the Regions I–III to be illustrated in a different

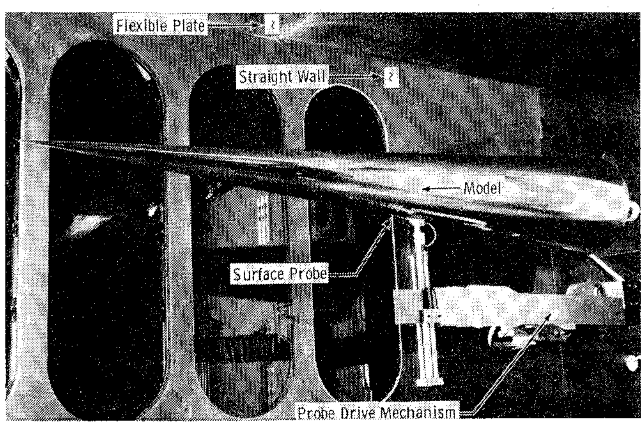

Fig. 3 VKF-Tunnel A cone model installation.

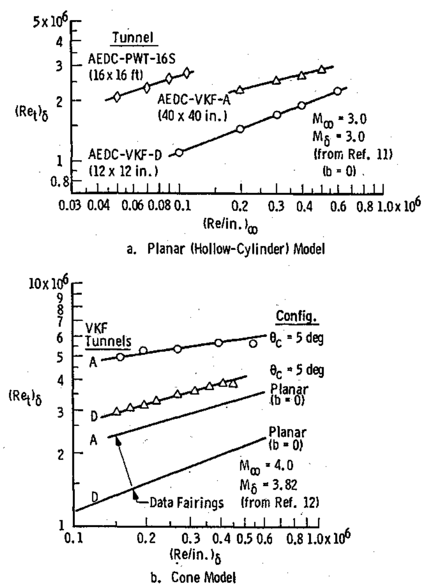

Fig. 4 Variation of adiabatic smooth wall transition. Reynolds number with tunnel size.

form. Also, data presented in this form are convenient for comparison with the results of van Driest.[2–4] Once again the large variation of the smooth cone $(Re_t)_\delta$ values with tunnel size is demonstrated. The data also show that a significant difference in trip effectiveness in Regions I-A and II exists. This avenue will be pursued in the following sections.

IV. Trip Correlations

van Driest et al.[2–4] proceeded with a systematic experimental and analytical program that resulted in a spherical roughness correlation of "effective" point Reynolds numbers that is applicable to both planar models and sharp cones. This correlation incorporated the effects of surface Mach number from zero to approximately four and the effects of surface cooling obtainable in standard high-speed tunnels.

Results from the present investigation are compared in Fig. 11 with results of van Driest and Blumer.[3,4] The agreement is considered good, and the author feels that these results provide additional confirmation to the conclusions of Refs. 3 and 4. It is to be specifically noted that all the data presented in Fig. 11 are based on the "effective point" concept. The "effective point" roughness criteria are based on the hypothesis that all x_t locations upstream of the "effective point" location (Region III) are trip dominated and controlled as $(Re/in.)_\infty$ is varied, but downstream of this location (Regions II and I-A) the x_t values are significantly influenced by the combined effects of surface trip disturbances and freestream disturbances (whatever the mode).

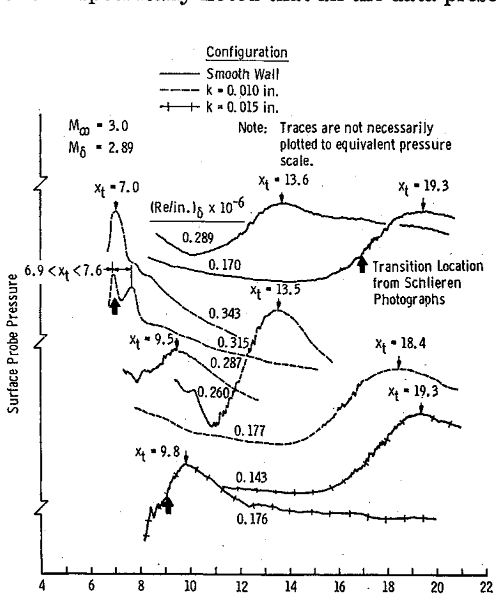

Fig. 5 Surface probe transition profiles, M_∞ = 3.0, VKF-Tunnel D.

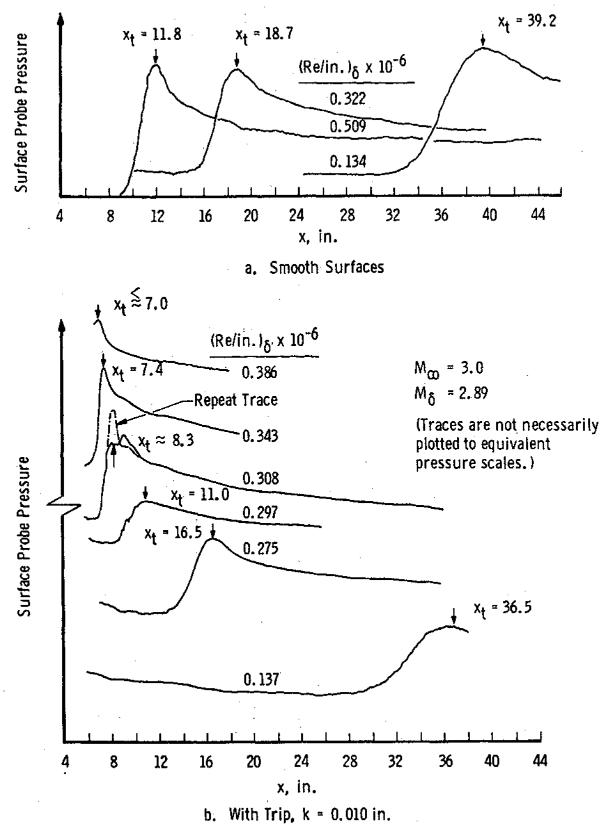

Fig. 6 Surface probe transition profiles, M_∞ = 3.0, VKF-Tunnel A.

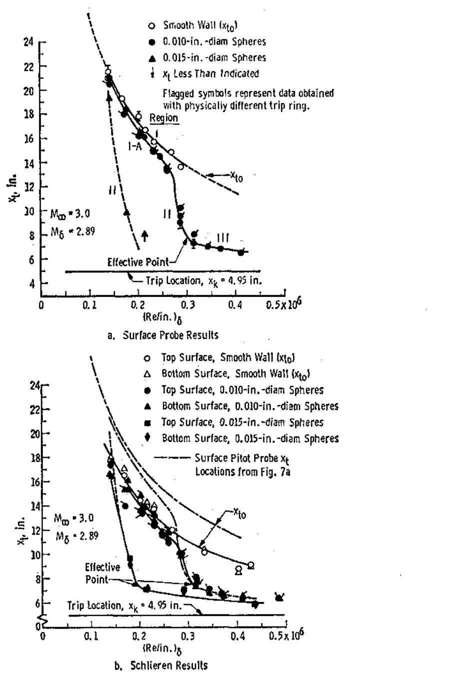

Fig. 7 Effect of spherical roughness on transition location, M_δ = 2.89, VKF-Tunnel D.

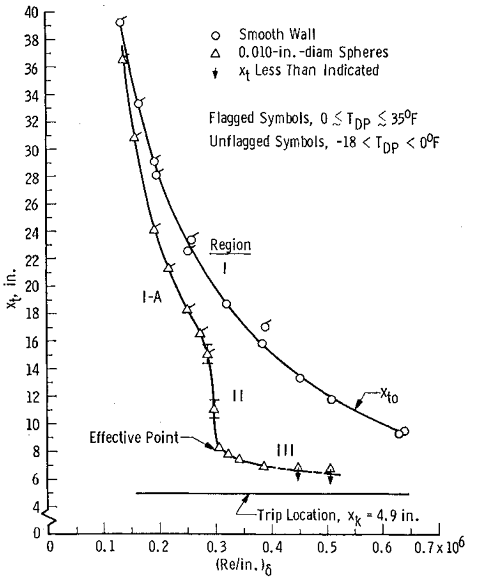

Fig. 8 Effect of spherical roughness on transition location, M_δ = 2.89, VKF-Tunnel A.

When evaluating the comparison between the two independent sets of data, it should be kept in mind that the test models, trip sizes, and general test conditions were nearly identical. Therefore, agreement would be expected, providing the data were valid and all extraneous sources of error had been eliminated (or were identical) in both experiments. It is nevertheless reassuring to observe that different investigators can produce reproducible and repeatable results even in the very uncertain area of boundary-layer tripping. In addition to the agreement exhibited in Fig. 11, direct comparison of the tripped transition locations [x_t vs $(Re/\text{in.})_\delta$] in the 12-in. Tunnel D data with the 12-in. JPL tunnel results in Ref. 2 also shows reasonable agreement. It was additionally encouraging to establish that the traversing probe produced results comparable in quality to the surface temperature and the magnified Schlieren technique of Refs. 2–4.

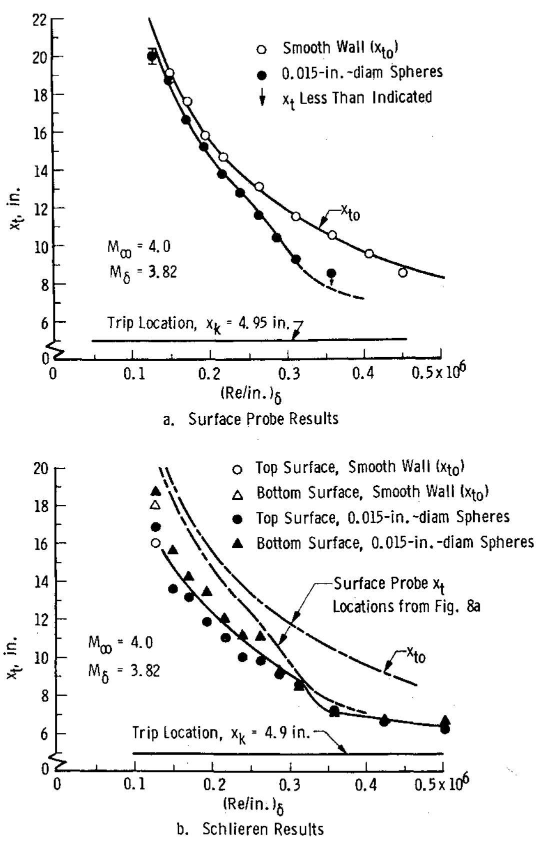

Fig. 9 Effect of spherical roughness on transition location, M_δ = 3.82, VKF-Tunnel D.

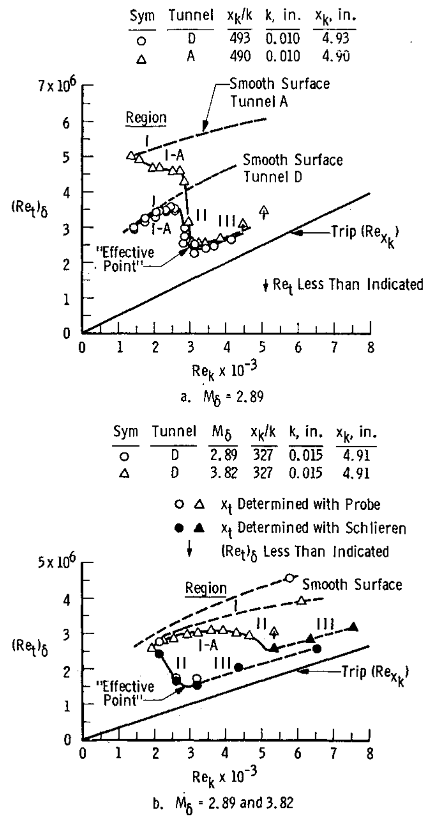

Fig. 10 Variation of transition Reynolds numbers with trip Reynolds number for M_δ = 2.89 and 3.82, Tunnels D and A.

The trip correlation of Potter and Whitfield first appeared in its present form in Ref. 6 and was further extended to include higher Mach numbers in Refs. 7 and 8. To the author's knowledge this correlation is the most comprehensive of any published to date. The correlation incorporates, as did the

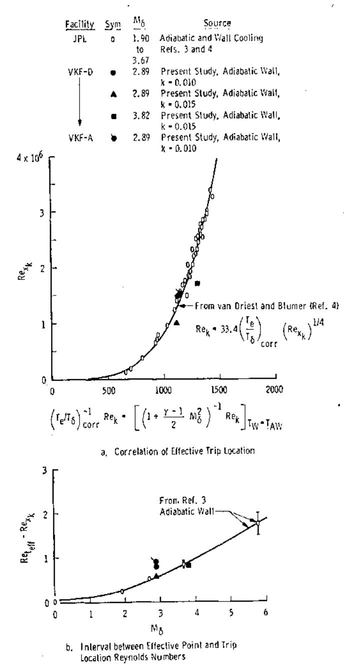

Fig. 11 Correlation of tripped results using the methods of van Driest-Blumer.

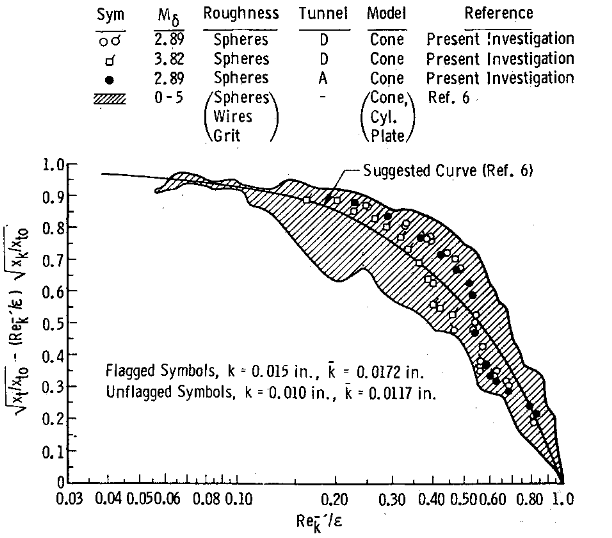

Fig. 12 Correlation of tripped results using the method of Potter-Whitfield.

correlation of van Driest and Blumer, the effects of heat transfer obtainable in standard high-speed tunnels and Mach number, and is applicable to both planar models and sharp cones. However, the local Mach number range extends to approximately 10, and the correlation provides values of the trip size required to locate transition anywhere between the undisturbed smooth surface transition location and the trip position. The correlation also predicts the effectiveness of single two-dimensional surface wires in addition to single rows of spheres.

Data from the present investigation are presented in Fig. 12 using the correlation of Potter and Whitfield. The current data are seen to lie within the data band used in Ref. 6 to establish a suggested curve. One observation is that the suggested correlation curve will not predict the "effective point" or "knee" location of the present data. However, a significant result to note is that the correlating parameters provided a nearly identical collapse of the $M_\delta = 2.89$, $k = 0.010$ in. tripped data from the VKF Tunnels A and D. It is to be remembered that the only difference in these data (in terms of tunnel conditions and model and trip geometry) is the large

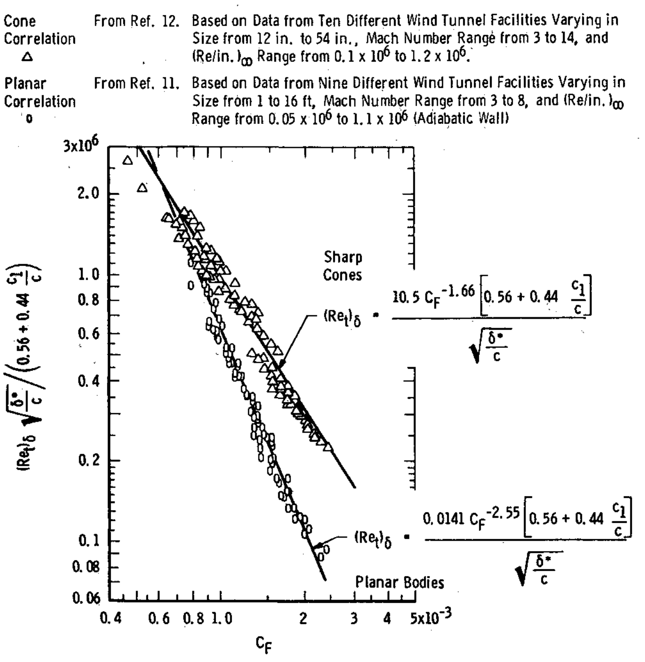

Fig. 13 Correlation of transition Reynolds numbers.

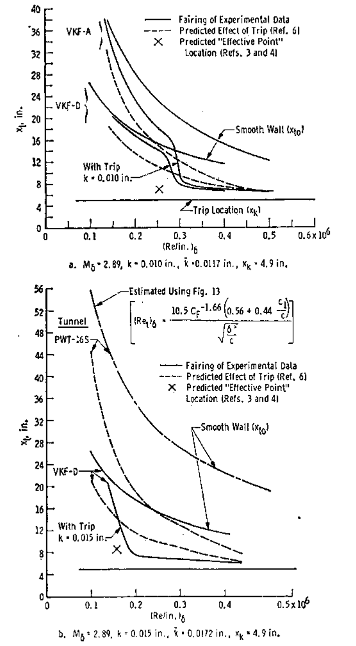

Fig. 14 Comparisons of correlation predictions with experimental results.

difference in the smooth wall x_{t_0} locations. A tentative conclusion to be inferred from these results is that the x_{t_0} normalizing parameter in the Potter-Whitfield correlation will allow data in the supersonic Mach number regime ($M_\delta < 5$) from various size tunnels to be correlated.

Application of the Potter-Whitfield correlation parameters requires that the Mach number ($M_{\bar{k}}$) in undisturbed laminar flow at height $\bar{k}$ be known. Values of $M_{\bar{k}}$ were obtained using the flat plate similarity parameter [$\eta = (y/x_k)(Re_{x\bar{k}})^{1/2}$] divided by the $(3)^{1/2}$ in conjunction with the results of Ref. 13. Values of $M_{\bar{k}}$ for sharp cones can also be obtained directly from graphical results presented in Ref. 14.

In determining the boundary-layer thickness it has been assumed that the entropy layer generated by the bow shock has been completely "swallowed" by the boundary layer, and the cone acts as an "aerodynamically" sharp cone. This is a reasonable assumption, since the ratio of the trip location distance to the nose radius is approximately 2000.

V. Method of Estimating Smooth Surface $(Re_t)_\delta$ Values

Inspection of the correlation parameters of Potter and Whitfield in Fig. 12 reveals that the smooth surface value of the transition location must be known. The studies by Pate and Schueler[11] and Pate[12] have shown that the transition location on models in supersonic and hypersonic tunnels $3 \lesssim M_\infty \lesssim 8$ will vary with tunnel size in accordance with the radiated aerodynamic noise disturbance philosophy. A unique correlation of transition Reynolds numbers as a function of the aerodynamic noise parameters (δ^* and C_F) and the tunnel circumference (c) for both planar and sharp cones was published in Refs. 11 and 12. These correlations are shown in Fig. 13. Therefore, when transition locations for $3 \lesssim M_\infty \lesssim 10$ are desired for application in the Potter-Whitfield trip correlation, and measured values of x_{t_0} from the facility under consideration are not available, the correlations presented in Fig. 13 can be utilized to obtain estimated smooth wall $(Re_t)_\delta$ values for either flat plates (or hollow cylinders) and sharp slender cones.

It is to be noted that the correlations presented in Fig. 13 are applicable only to "conventional" wind tunnels having a turbulent boundary layer on the tunnel walls for $3 \lesssim M_\infty \lesssim$

10 and cannot be used to estimate $(Re_t)_\delta$ values in ballistic ranges or in atmospheric flight.

VI. Comparisons between Experiments and Predictions

Direct comparisons between the experimental data and the estimated x_t locations using the correlation of Potter and Whitfield[6] for the specified spherical roughness heights ($\bar{k}$) of 0.0117 and 0.0172 in. for a local cone surface Mach number of 2.89 are provided in Fig. 14. The methods of van Driest and Blumer, as presented in Fig. 11, enable the "effective point" location to be estimated. Estimates of tripped x_t locations to be expected in very large supersonic tunnels, such as the AEDC-PWT-16S tunnel (16 ft × 16 ft), are included in addition to the experimental data from the AEDC-VKF Tunnels D and A. The x_{t_o} values used in the trip correlations are those values shown in Fig. 14.

It is evident that when the Potter-Whitfield suggested curve from Fig. 12 is used, the "effective point" or "knee" location is not predicted. This deficiency could, of course, be eliminated if a different "suggested curve" were used. J. L. Potter[15] told the author that he originally looked for evidence that the suggested curve in Fig. 12 should exhibit a "knee" or asymptotic approach to the abscissa in the region $x_t \rightarrow x_k$, but could not justify presenting the curve in that form (on the basis of the data used in Ref. 6) even though he thought some change in shape near the right side of the curve had plausibility. However, upon inspection of the data band in Fig. 12 it is not immediately evident that a different "suggested curve" would be justified. The disagreement in terms of physical distance (in.) also increases with tunnel size. The method of van Driest and Blumer predicts the "effective point" x_t location quite adequately. The agreement between predictions and measurements using both methods is considered fairly good and within the correlation scatter of the methods (e.g., see Figs. 11 and 12).

Although perhaps somewhat limited in scope, the present study has provided results which are considered to be significant. Also, since the trip correlations of Refs. 3, 4, and 6 apply to planar and conical geometries and since the freestream radiated noise disturbances affect transition on planar and conical bodies in a similar manner (Refs. 11 and 12), the results of this study can also be expected to apply to spherical roughness on two-dimensional surfaces. The test Mach numbers of 3 and 4, although not covering a large Mach number range, are Mach numbers of prime interest in many wind tunnel test programs. The data in Figs. 10 and 14 indicate that the transition location in Region III between the trip and the effective point location is a weak function and in Region II and IA a much stronger function of the freestream disturbance level. The data also indicate that the Potter-Whitfield correlation through the use of the x_{t_o} term successfully collapsed the tripped data. The magnitude of discrepancies between predicted and wind-tunnel results which the user must expect until a Potter-Whitfield type correlation with a bend or "knee" becomes available is suggested by the differences between the solid and dotted (tripped) curves in Fig. 14. The present data support the "effective point" criteria proposed by van Driest and Blumer and suggest this location will be valid for all sizes of supersonic ($M_\infty \lesssim 5$) wind tunnels.

VII. Concluding Remarks

Boundary-layer transition experiments conducted in the AEDC-VKF 12- and 40-in. wind tunnels at Mach numbers 3 and 4 on a 10° total-angle, adiabatic wall, sharp cone have shown that the absolute effectiveness of spherical roughness can be influenced by the freestream disturbances (aerodynamic noise) present. It is therefore concluded that to a significant degree the tripped transition location at supersonic speeds ($M_\infty \approx 3$ to 5) is dependent on the tunnel size or, more precisely, the freestream disturbances. Thus, it appears appropriate to relate roughness effects to the smooth wall transition location (x_{t_o}), as done by Potter and Whitfield, when attempting to normalize tunnel flow effects. The trip correlation parameters developed by Potter and Whitfield correlated the tripped transition data obtained in two significantly different size supersonic tunnels having significantly different x_{t_o} values, but the correlation in its present form does not predict an effective point or "knee" location. These studies confirmed the "effective point" criteria proposed by van Driest et al., and verified that the "effective point" is trip disturbance dominated and essentially independent of the tunnel disturbance levels (x_{t_o} location) at supersonic ($M_\infty \approx 3$ to 5) speeds.

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